FACE RECOGNITION BY LINEAR DISCRIMINANT ANALYSIS

SUMAN KUMAR BHATTACHARYYA¹, KUMAR RAHUL²

¹,²Computer Science and Engineering Department, Indian School of Mines, Dhanbad, Jharkhand-826004, India
E-mail: sumanb_it33@yahoo.co.in, kumarrahul.icfai@gmail.com

Abstract- Linear Discriminant Analysis (LDA) has been successfully applied to face recognition which is based on a linear projection from the image space to a low dimensional space by maximizing the between class scatter and minimizing the within-class scatter. LDA allows objective evaluation of the significance of visual information in different features of the face for identifying the human face. The LDA also provides us with a small set of features that carry the most relevant information for classification purposes. LDA method overcomes the limitation of Principle Component Analysis method by applying the linear discriminant criterion. This criterion tries to maximize the ratio of determinant of the between-class scatter matrix of the projected samples to the determinant of the within-class scatter matrix of the projected samples. Linear discriminant groups the images of the same class and separate images of different classes. Here to identify an input test image, the projected test image is compared to each projected training, and the test image is identified as the closest training image. The experiments in this paper we present to use LDA for face recognition. The experiments in this paper are performed with the ORL face database. The experimental results show that the correct recognition rate of this method is higher than that of previous techniques.

Keywords- Face recognition, Linear Discriminant Analysis, Class separation using LDA, Algorithm used in LDA approach, Experimental result.

I. INTRODUCTION

Face recognition system is a computer application for automatically identify or verifying a person from a digital image or video frame from a video source. Facial recognition system typically used in security system. In this system automatically searching of faces from the face databases, typically resulting in a group of facial images ranked by computer evaluated similarity. Some facial recognition algorithm identifies faces by extracting landmarks, or features from an image of the subject face. For example, face recognition algorithm may analyze the relative position, size, shape of the eyes, nose cheekbones and jaw to recognize faces.

Linear Discriminant analysis explicitly attempts to model the difference between the classes of data. LDA is a powerful face recognition technique that overcomes the limitation of Principle component analysis technique by applying the linear discriminant criterion. This criterion tries to maximize the ratio of the determinant of the between-class scatter matrix of the projected samples to the determinant of the within-class scatter matrix of the projected samples. Linear discriminant group images of the same class and separates images of the different classes of the images.

Discriminant analysis can be used only for classification not for regression. The target variable may have two or more categories. Images are projected from two dimensional spaces to c dimensional space, where c is the number of classes of the images. To identify an input test image, the projected test image is compared to each projected training image, and the test image is identified as the closest training image. The LDA method tries to find the subspace that discriminates different face classes. The within-class scatter matrix is also called intra-personal means variation in appearance of the same individual due to different lighting and face expression. The between-class scatter matrix also called the extra personal represents variation in appearance due to difference in identity. Linear discriminant methods group images of the same classes and separates images of the different classes. To identify an input test image, the projected test image is compared to each projected training image, and the test image is identified as the closest training image.

To explain discriminant analysis, here we consider a classification involving two target categories and two predictor variables.

The following figure shows a plot of the two categories with the two predictor’s orthogonal axes:

![Figure 1. Plot of two categories](image-url)
Linear discriminant analysis finds a linear transformation (discriminant function) of the two predictors, X and Y that yields a new set of transformed values that provides a more accurate discrimination than either predictor alone:

\[
\text{Transformed Target} = C_1 \cdot X + C_2 \cdot Y
\]

The following figure shows the partitioning done using the transformation function:

Maximizing the between class scatter matrix while minimizing the within-class scatter matrix, a transformation function is found that maximizes the ratio of between-class variance to within-class variance and find a good class separation as illustrated as follows:

**Good class separation**

Figure 3. Class Separations in LDA

**II. ALGORITHM USED IN LDA**

In Linear discriminant analysis we provide the following steps to discriminate the input images:

**Step-1**
We need a training set composed of a relatively large group of subjects with diverse facial characteristics. The appropriate selection of the training set directly determines the validity of the final results. The database should contain several examples of face images for each subject in the training set and at least one example in the test set. These examples should represent different frontal views of subjects with minor variations in view angle. They should also include different facial expressions, different lighting and background conditions, and examples with and without glasses. It is assumed that all images are already normalized to \( m \times n \) arrays and that they contain only the face regions and not much of the subjects’ bodies.

**Step-2**
For each image and sub-image, starting with the two dimensional \( m \times n \) array of intensity values \( I(x, y) \), we construct the vector expansion \( \Phi \in \mathbb{R}^{m \times n} \). This vector corresponds to the initial representation of the face. Thus the set of all faces in the feature space is treated as a high-dimensional vector space.

**Step-3**
By defining all instances of the same person’s face as being in one class and the faces of different subjects as being in different classes for all subjects in the training set, we establish a framework for performing a cluster separation analysis in the feature space. Also, having labeled all instances in the training set and having defined all the classes, we compute the within-class and between-class scatter matrices.

Now with-in class scatter matrix ‘\( S_w \)’ and the between class scatter matrix ‘\( S_b \)’ are defined as follows:

\[
S_w = \sum_{c} \sum_{i=1}^{N_j} (\Gamma_i - \mu_j)(\Gamma_i - \mu_j)^T \quad \text{-------- (1)}
\]

Where, \( \Gamma_i \), the \( i \)th samples of class \( j \), \( \mu_j \) is the mean of class \( j \), \( c \) is the number of classes, \( N_j \) is the number of samples in class \( j \).

\[
S_b = \sum_{j=1}^{c} (\mu_j - \mu)(\mu_j - \mu)^T \quad \text{------------------------ (2)}
\]

Where, \( \mu \) represents the mean of all classes.

\[
W = \arg \max \quad mod \left[ \frac{W^T \cdot S_b \cdot W}{W^T \cdot S_w \cdot W} \right]
\]

Then the subspace for LDA is spanned by a set of vectors \( W=[W_1, W_2, \ldots, W_d] \), Satisfying

\[
W = \arg \max \quad mod \left[ \frac{W^T \cdot S_b \cdot W}{W^T \cdot S_w \cdot W} \right] \quad \text{-------- (3)}
\]

The with class scatter matrix represents how face images are distributed closely within classes and between class scatter matrix describes how classes are separated from each other. When face images are projected into the discriminant vector \( W \).

Face images should be distributed closely within classes and should be separated between classes, as much as possible. In other words, these discriminant vectors minimize the denominator and maximize the numerator in equation (3). \( W \) can therefore be constructed by the eigenvectors of \( S_w^{-1} \cdot S_b \). PCA tries to generalize the input data to extract the features and
LDA tries to discriminate the input data by dimension reduction.

The testing phase of the Linear Discriminant Analysis is as shown as in figure below:

![Figure 4. Step of testing phase of LDA](image)

### III. APPLICATION OF LDA

- Linear discriminant analysis techniques used in statistics, pattern recognition and machine learning to find a linear combination of features which characterized or separates two or more classes of objects or events.

- In LDA resulting combination may be used as a linear classifier or more commonly for dimensionality reduction.

### IV. DATABASE USED

Here database of face used AT & T “The Database of Faces” which is also known as “The ORL Database of Faces”. There are ten different images of each of 40 distinct subjects. For some subjects, the images were taken at different times, varying the lighting, facial expressions (open or closed eyes, smiling or not smiling) and facial details such as glasses or without glasses. All the images were taken against a dark homogeneous background with the subjects in an upright, frontal position.

All the files are in PGM format. The size of each image is $92 \times 112$ pixels, with 256 grey levels per pixel. The images are organized in 40 directories (one for each subject), which have names of the form sx, where $x$ indicates the subject number (between 1 and 40). In each of these directories, there are ten different images of that subject, which have names of the form Y.pgm, where Y is the image number for the subject between 1 and 10.

![Figure 5. ORL Database](image)
**B. Training Data Sets:**
After forming the test data set we form training data set from the ORL database by extracting 40 images which are present in the test data set. So for the each person we have now 9 different images in all the 40 directories. Total 360 images present in our training dataset. All the images are converted into bitmap (.bmp) image format.

**V. EXPERIMENTAL RESULT**
The well known face database, the ORL database used to demonstrate the effectiveness of the LDA approach.

Here we test all our test images with image number 1 to 40 images. The entire test image compares with the image of our Training Data Set using linear discriminant techniques. We get true result 37 times, means 37 images are recognized correctly by the face images of correct person. And we get false result 3 times, means 3 images are recognized by face images of wrong person. The result of our experiment is as follows:

<table>
<thead>
<tr>
<th>Test image number</th>
<th>Right result</th>
<th>Wrong result</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>T</td>
<td>-</td>
</tr>
<tr>
<td>2</td>
<td>T</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>T</td>
<td>-</td>
</tr>
<tr>
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<td>T</td>
<td>-</td>
</tr>
<tr>
<td>19</td>
<td>-</td>
<td>F</td>
</tr>
</tbody>
</table>

Table 1. Result of experiment Wrong Result:

<table>
<thead>
<tr>
<th>Sl No</th>
<th>Test Image</th>
<th>Equivalent Image</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td><img src="image1.png" alt="Image" /></td>
<td><img src="image2.png" alt="Image" /></td>
</tr>
<tr>
<td>2</td>
<td><img src="image3.png" alt="Image" /></td>
<td><img src="image4.png" alt="Image" /></td>
</tr>
<tr>
<td>3</td>
<td><img src="image5.png" alt="Image" /></td>
<td><img src="image6.png" alt="Image" /></td>
</tr>
</tbody>
</table>

Table 2. Result where not recognized correctly

Therefore in our experiment 37 images of human face recognized correctly and we get the appropriate matched image in output. And the total number of tested image here is 40. So the true positive rate is \((= 37 \div 40) 0.925\).

Similarly in our experiment 3 images of tested images can not recognized correct person by his face image and in this 3 case we get the output with image of different person with much similarities. So the true negative rate is \((= 3 \div 40) 0.075\).
### Table 3. Performance of LDA

<table>
<thead>
<tr>
<th>Number of Test</th>
<th>Correctly recognized</th>
<th>Wrongly recognized</th>
<th>Accuracy (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>37</td>
<td>3</td>
<td>92.5</td>
</tr>
</tbody>
</table>

**VI. CONCLUSION**

Linear Discriminant Analysis method has been successfully applied to face recognition which is based on a linear projection from the image space to a low dimensional space. But the major drawback of applying LDA is that it may encounter the small sample size problem. When the small sample size problem occurs, the within-class scatter matrix becomes singular. Since the within-class scatter of all the samples is zero in the null space of $S_w$, the projection vector that can satisfy the objective of an LDA process is the one that can maximize the between-class scatter.

But face image data distribution in practice is highly complex because of illumination, facial expression and pose variation. The kernel technique is used to project the input data into an implicit space called feature space by nonlinear kernel mapping. Therefore kernel trick is used taking input space and after that LDA performed in this feature space, thus a non linear discriminant can be yielded in the input data.

**REFERENCES**


