EXPERIMENTAL VALIDATION AND DATA ACQUISITION FOR HYPER ELASTIC MATERIAL MODELS IN FINITE ELEMENT ANALYSIS

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Abstract—This paper presents the theory, experiment setups and solution implementation of Hyperelastic Material Models in Finite Element Analysis to provide the description of material behavior that matches the conditions the product sees in real life. This can be a complex matter because the real life scenario may have the product responding simultaneously to a multiplicity of conditions such as rate, temperature and the environment. Physical testing of elastomers for the purpose of fitting material models in finite element analysis requires experiments like uniaxial tension, biaxial tension, volumetric compression in multiple states of strain under carefully considered loading conditions.

Keywords: Finite Element Analysis, Hyperelastic Material Models, Elastomers, Volumetric compression test, Validation and Data Acquisition

I. INTRODUCTION

Finite element analysis (FEA), including pre- and post processing, is seeing wider use despite being hampered by excessive setup time and computational requirements. 2-D applications outnumber 3-D applications due to scaling laws. Various engineers have been attributed to being the father of FEA, e.g., Courant (1943). However, the arrival of the digital computer especially in the aircraft industry led to a rapid interest in activity in the Boeing Corporation in the early 1950's. The Structural Dynamics Unit, led by M J Turner, formulated the method in 1954 and published it in 1956. The North American B-70 bomber was the first production airplane designed using FEA. The World Trade Center in New York and the John Hancock Center in Chicago were the first buildings designed on the basis of FEA.

Advances in automatic mesh generation algorithms have the potential to increase use of FEA methods by an order of magnitude. The integration of manufacturing applications into systems will be paced by software, and will be slow and incremental.

The basic concept of finite element method is discretization of a structure into finite number of elements, connected at finite number of points called nodes. The material properties and the governing relationships are considered over these elements and expressed in terms of nodal displacement at nodes. An assembly process duly considering the loading and constraints results in a set of equations governing the structural response, which are established through the application of appropriate variation principle.

Solutions of these equations give the response of the structure. Selecting proper elements and subdividing the structure with large number of finite elements or by taking higher order elements can increase the accuracy of solution obtained by finite element method.

Hyperelastic models are used extensively in the finite element analysis of rubber and elastomers. These models need to be able to describe elastomeric behavior at large deformations and under different modes of deformation. In order to accomplish this daunting task, material models have been presented that can mathematically describe this behavior. There are several in common use today, notably, the Mooney-Rivlin, Ogden and ArudaBoyce.

II. HYPERELASTIC MATERIAL

Hyperelasticity refers to the materials, which can experience large elastic strain that is recoverable. Elastomer such as rubber and many other polymer materials fall in this category.

The microstructure of polymer solids consists of chain-like molecules. The chain backbone is mostly made up of the carbon atoms. The flexibility of polymer molecules allows different types of arrangement such as amorphous and semicrystalline polymers. As a result, the molecules possess a much less regular character than the metal crystals. The behavior of the elastomers are therefore very complex, on macroscopic scale, they usually behave elastically isotropic initially, and anisotropic at finite strain as the molecule chains tends to realign to loading direction.
Under essentially monotonically loading condition, however, a larger class of the elastomers can be approximated by an isotropic assumption, which has been historically popular in the modeling of the elastomers.

![Elastomer Material Behaviour](image1)

**Fig. 1 Elastomer Material Behaviour**

Elastomers are often modeled as hyperelastic. Elastomers (like rubber) typically have large strains (often some 100%) at small loads (means a very low modulus of elasticity). The material is nearly incompressible, so the Poisson’s ratio is very close to 0.5. Their loading and unloading stress-strain curve is not the same, depending on different influence factors (time, static or dynamic loading, frequency etc.). This viscous behavior is ignored if the hyperelastic material model is used for description.

![Linear Elastic Material like brittle steel or ceramic](image2)

**Fig. 2 Linear Elastic Material like brittle steel or ceramic**

The constitutive behaviors of hyperelastic materials are usually derived from the strain energy potentials. Also, hyperelastic materials generally have very small compressibility. This is often referred to incompressibility. The hyperelastic material models assume that materials response is isothermal. This assumption allows that the strain energy potentials are expressed in terms of strain invariants or principal stretch ratios. Except as otherwise indicated, the materials are also assumed to be nearly or purely incompressible. Material thermal expansion is always assumed to be isotropic.

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Force equals the spring rate times the deflection is one of the first equations an engineer meets. This equation is valid as long as the object remains linearly elastic. If one deflects the component twice as much, the force increases twice as much. If the structure yields, or large displacements occur, or the material for the spring has a non-linear stress-strain curve it may have a non-linear problem and not even know it.

### III. CHOOSING A MATERIAL MODEL

The hyperelastic material models present a number of options to aid in a best fit of the material data. Mooney-Rivlin model is the by far, the most common model in use today. It presents many advantages in terms of being able to handle the different kinds of behavior seen in rubbers. The ability to increase the number of modes permits the handling of large strain behaviors with some level of dexterity. The objective of any model development effort however, is to fit the data at as low a number of modes as possible.

The effect of weathering or the presence of oil, gasoline, body fluids or other chemicals can significantly affect the behavior of a material. The consequence of the environment is often unpredictable and may improve or adversely affect the performance of the product.

A classic example may be the rubber boot of an automotive CV joint that is simultaneously seeing large deformation, temperature, cyclic loading and oil or grease. To completely describe the material behavior would require a hyperelastic model on an oil soaked boot rubber over a range of temperatures with some consideration given to rate dependency. It becomes highly impractical to attempt to model all these situations. Accordingly, one often adopts a strategy that seeks to use the simplest acceptable model that achieves a reasonable approximation of the actual scenario. This strategy may be weighted to include a more detailed modeling of the greatest potential sources of failure. Careful thought given to material modeling at the start of the FEA project results in considerable savings in time, money and effort.

The form of the Mooney-Rivlin strain energy potential is

$$U = C_{12} (\bar{I}_1 - 3) + C_{02} (\bar{I}_2 - 3) + \frac{1}{D_1} (\bar{J} - 1)^2,$$

The three stretch invariants (because independent from the used coordinate system) of the characteristic equation are analog:
Experimental Validation and Data Acquisition for Hyper elastic Material Models in Finite Element Analysis

IV. EXPERIMENTAL VALIDATION AND DATA ACQUISITION

The FE experiments, covering several temperatures, strain rates, material models and combinations of input test data, generated a large quantity of results. These were plotted in the form of reaction force/length versus strain (gauge extension/bond thickness) in order to qualitatively compare the results with measured data. Statistical DOE methods were considered as an option for the analysis of this data. This type of analysis would require a single value as the ‘experimental’ result.

The most models share common test data input requirements. In general, stress and strain data sets developed by stretching the elastomer in several modes of deformation are required and “fitted” to sufficiently define the variables in the material models. Appropriate experimental loading sequences and realistic strain levels are needed to capture the elastomer behavior that applies in the analysis.

IV. NON-LINEAR ANALYSIS TYPES

A. Three major types of Non-linearity:
   1) A geometric non-linearity: It is due to large deformations or snap-through buckling.
   2) A material non-linearity: It is due to large strains, plasticity, hyperelasticity, creep, or viscoelasticity.
   3) A boundary non-linearity: It is due to the opening/closing of gaps, contact surfaces, and follower forces.

B. Types of Material Non-linearity

When stresses go beyond the linear elastic range, material behavior can be broadly divided into two classes:
   1) Time-independent behavior: Pasticity that is applicable to most ductile metals; non-linear elasticity that is applicable to rubber.
   2) Time-dependent behavior: creep, and visco-elasticity that are applicable to high-temperature uses; viscoelasticity that is applicable to elastomers and plastics.

Where C10, C01 and d are material constants.

If C01 = 0, we obtain a neo-Hookean solid, a special case of a Mooney–Rivlin solid.

The nominal or engineering strain is defined as the change in length divided by the original length:

\[ \varepsilon = \frac{l_1 - l_0}{l_0} = \frac{\Delta l}{l_0} \]

The stretch ratio l now is another fundamental quantity to describe material deformation. It is defined as the current length divided by the original length:

\[ \lambda = \frac{l_1}{l_0} = \frac{l_1 - l_0 + l_0}{l_0} = \varepsilon + 1 \]

Analog to the three principal strains, we obtain from the principal axis transformation the three principal stretch ratios \( \lambda_1, \lambda_2, \lambda_3 \).

Figure 3 Final Data Set for Input into a Curve Fitter

a. Uniaxial Tension Experiment setup

This is the classical uniaxial tension rod mounted into a tensile testing machine. The strain must of course be measured in the thinner area of the test rod, for example by optical scanning (video extensometry); the thicker parts of the tension rod which are clamped must not be taken into account.
Experimental Validation and Data Acquisition for Hyper elastic Material Models in Finite Element Analysis

**c. Volumetric Compression Test setup**

A volumetric test setup like this compresses a cylindrical elastomer specimen constrained in a stiff fixture. The actual displacement during compression is very small and great care must be taken to measure only the specimen compliance and not the stiffness of the instrument itself. The initial slope of the resulting stress-strain function is the bulk modulus. This value is typically 2-3 orders of magnitude greater than the shear modulus for dense elastomers.

**d. Physical Testing and Validation**

Although the experiments are performed separately and the strain states are different, data from all of the individual experiments is used as a set. This means that the specimens used for each of the experiments must be of the same material. This may seem obvious but if the specimens are specially molded to accommodate the differing instrument clamps for different experiments, it is possible that the material processing parameters may cause material variations from test to test. While it is reasonable to assume that variation exists in the production environment and that we can never really get the exact material properties every time, it is not acceptable to have this same variation within the data set. The data represents a “snapshot” in time. If even slight variation exists between experiments, a physically impossible material model may be developed in the analysis software. The best way to avoid this problem is to cut specimens for simple tension, pure shear and equal biaxial extension from the same slab of material. The loading conditions, strain levels and straining rates should also be developed considering the inter-relationship between tests.

FEA Engineer requires support from test department for the activities namely Data acquisition for input(boundary conditions), Validation of FEA results and Field/laboratory failure reports.
Experimental Validation and Data Acquisition for Hyperelastic Material Models in Finite Element Analysis

e. Limitations of Hyperelastic Material Models

Most material models in commercially available finite element analysis codes allow the analyst to describe only a subset of the structural properties of elastomers. This discussion revolves around hyperelastic material models such as the Mooney-Rivlin and Ogden formulations and relates to those issues which effect testing.

- The stress strain functions in the model are stable. They do not change with repetitive loading. The material model does not differentiate between a 1st time strain and a 100th time straining of the part under analysis.
- There is no provision to alter the stress strain description in the material model based on the maximum strains experienced.
- The stress strain function is fully reversible so that increasing strains and decreasing strains use the same stress strain function. Loading and unloading the part under analysis is the same.
- The models treat the material as perfectly elastic meaning that there is no provision for permanent strain deformation. Zero stress is always zero strain.

f. Hints for Elastomeric FEA

- Stay away from triangular elements. Elements with 2 displacement BC will have only 1 degree of freedom due to incompressibility.
- Low order elements converge easiest. 4-node brick works well.
- Sliding contact may require non-symmetric stiffness matrices for large friction coefficients.
- Watch corners for element distortion.
- u-P element formulation is most stable.
- Check for stability of material models.

V. CONCLUSION

Finite Element analysis helps in accurating design and development of products by minimizing number of physical tests, there by reducing cost of prototyping and testing. Here an attempt is made to throw a light on Experimental Validation and Data Acquisition for Hyperelastic Material Models which present new challenges in automotive, aerospace, consumer goods and industrial products.

REFERENCES